Semi-Lagrangian variable-resolution numerical weather prediction model and its further development

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Outline

- Brief history and description of SL-AV model
- Results from quasioperational tests at Hydrometcentre for Dec 2004-Aug 2005
- Further developments:

  - Reduced lat-lon grid

  - Precipitation forecasts

  - Development of assimilation for soil variables
- Development of 2D nonhydrostatic dynamics

Why to use lat-lon grid?

- The lat-lon grid possesses convenience of discrete formulation and coding

- The disadvantages of a regular lat-lon grid can be potentially overcome with the use of a reduced lat-lon grid

SL-AV model
(semi-Lagrangian absolute vorticity)

- **Shallow water constant-resolution version demonstrated the accuracy of a spectral model for most complicated tests from the standard test set**

  (JCP 2002 v. 179, 180-200)

- **3D constant-resolution version (Russian Meteorology and Hydrology, 2001, N4) passed quasioperational tests at RHMC**

- **3D dynamical core passed Held-Suarez test**

Features

- **Constant resolution version** – 0.9x0.72 degrees (lon x lat), 28 sigma levels (1.40625x1.125 for seasonal forecasts)

- **Variable resolution version** – 0.5625° lon, lat resolution varying between ~30 and 70 km, 28 levels

- Possibility to use configuration with rotated pole in future ('advected' Coriolis term)

- **Parameterizations** from operational Meteo-France ARPEGE/IFS model with minor modifications

Features of dynamics

- **Semi-Lagrangian scheme** – SETTLS (Hortal, QJRMS 2003)

- **Semi-implicit scheme** – follows (Bates et al, MWR 1993) but with trapezoidal rather than midpoint rule in hydrostatic equation

- **4th-order differencing formulae** (compact and explicit) for horizontal derivatives

- **Direct FFT solvers** for semi-implicit scheme, U-V reconstruction, and 4th order horizontal diffusion

Parameterizations of subgrid-scale processes

Developed for French operational ARPEGE/IFS model

- **Short- and longwave radiation** (Geleyn, MWR 1992 modif.)

- **Deep convection** – modified Bougeault mass-flux scheme (MWR 1985), includes downdratfs, also the momentum change (Gregory, Kershaw, QJ 1997)

- **Planetary boundary layer** - modified (Louis et al, ECMWF procs 1982). "Interactive" mixing length has just been implemented.

- **Gravity wave drag** – includes mountain anisotropy, resonance trapping and lift effects (Geleyn et al 2004).

- Simple **surface scheme** (ISBA scheme is under tuning)

Parallel implementation for version 0.225ºx0.18ºx28
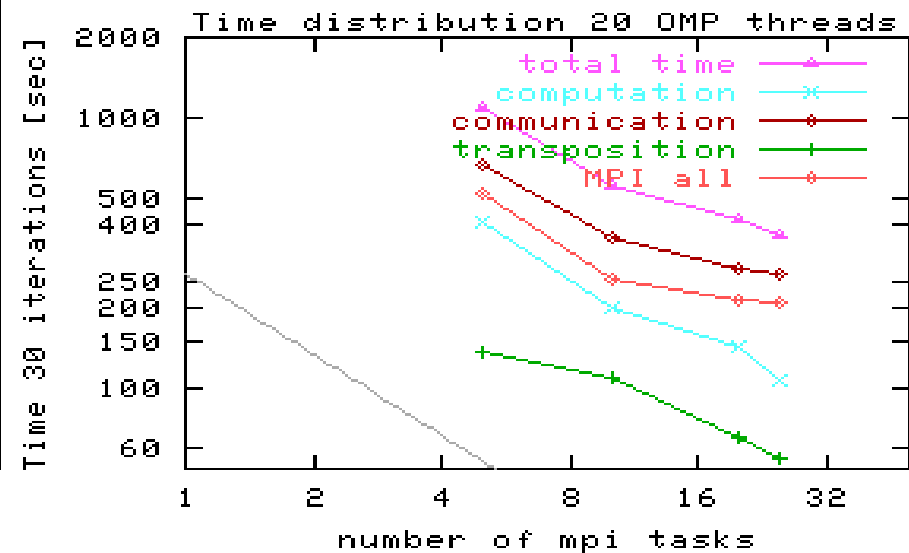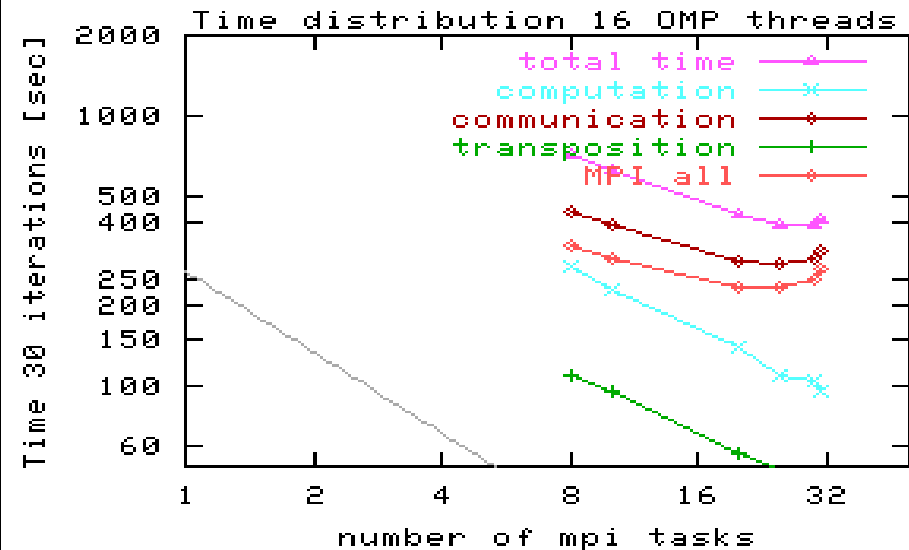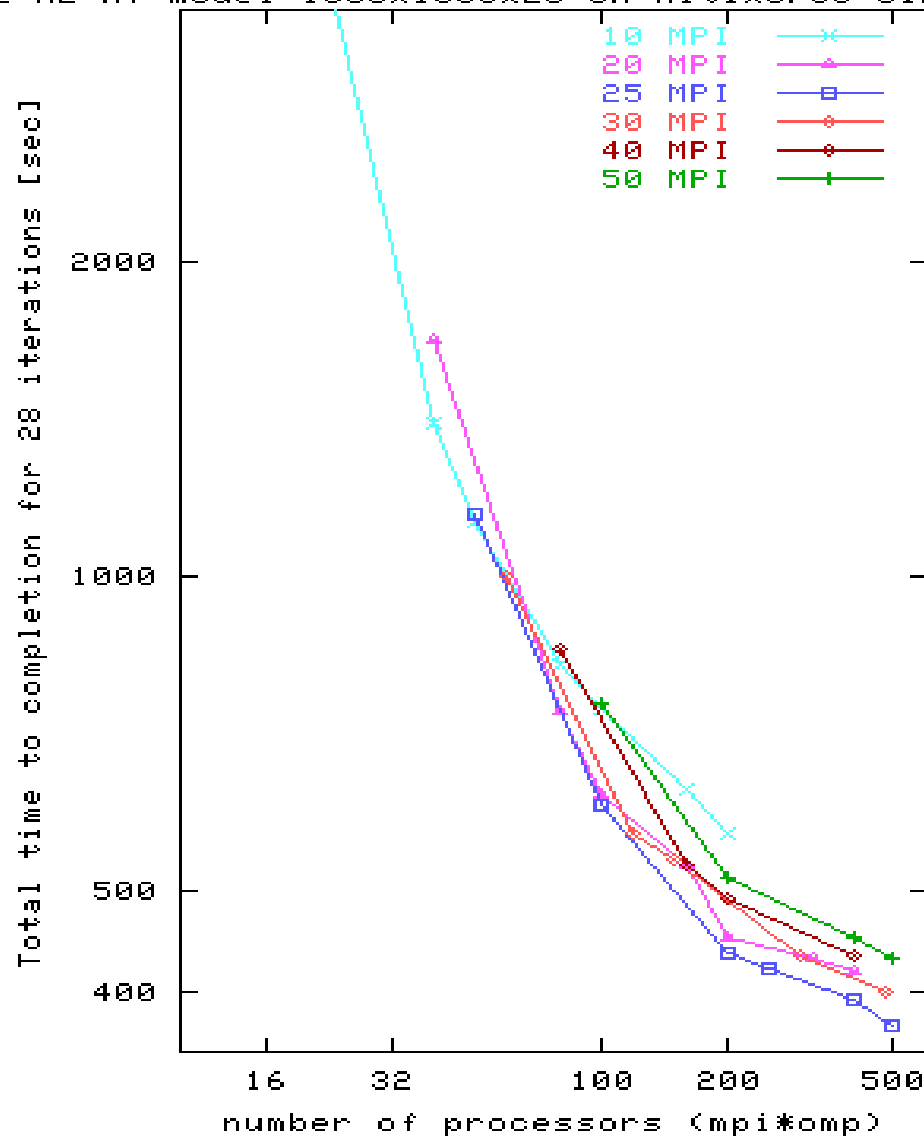
Extension to the case of variable resolution in latitude

- Discrete coordinate transformation (given as a sequence of local map factors), subject to smoothness and ratio constraints. This requires very moderate changes in the constant resolution code.

- Some changes in the semi-Lagrangian advection - interpolations and search of trajectories on a variable mesh.

- Described in *Tolstykh, Russ. J. Num. An. & Math. Mod., 2003.*

The variable grid strategy is limited to the relatively short-range forecasts, since for medium range forecasts, the high resolution region will come under influence of weather systems that at initial time are far away, and hence are poorly resolved in the analysis.

Problem in the variable-resolution version of the model

– clustering of grid points near the poles due to convergence of meridians on the latitude-longitude grid, especially in the high-resolution case.

This drawback leads to problems in use of parallel iterative solvers, calculation of grid-point humidity convergence needed for deep convection parameterization, and also to expenses on calculation in "wasted" grid points. The situation aggravates when one uses the variable resolution in latitude.

It is necessary to work on reduced grid implementation

Idea: The accuracy of the SL scheme substantially depends on the interpolation procedure

Numerical approach

$$f(d) = e^{-\mu d^2}, \quad \mu = -\frac{4\ln(\delta)}{a^2\beta^2} \quad \text{symmetric feature with}$$

the center at $(0, \phi_0)$ and the angle size β

$$\Phi(f, \phi_0, \beta, \{n_{\lambda,k}\}_{k=1}^{n_\phi}) \quad \text{is the r. m. s. interpolation error}$$
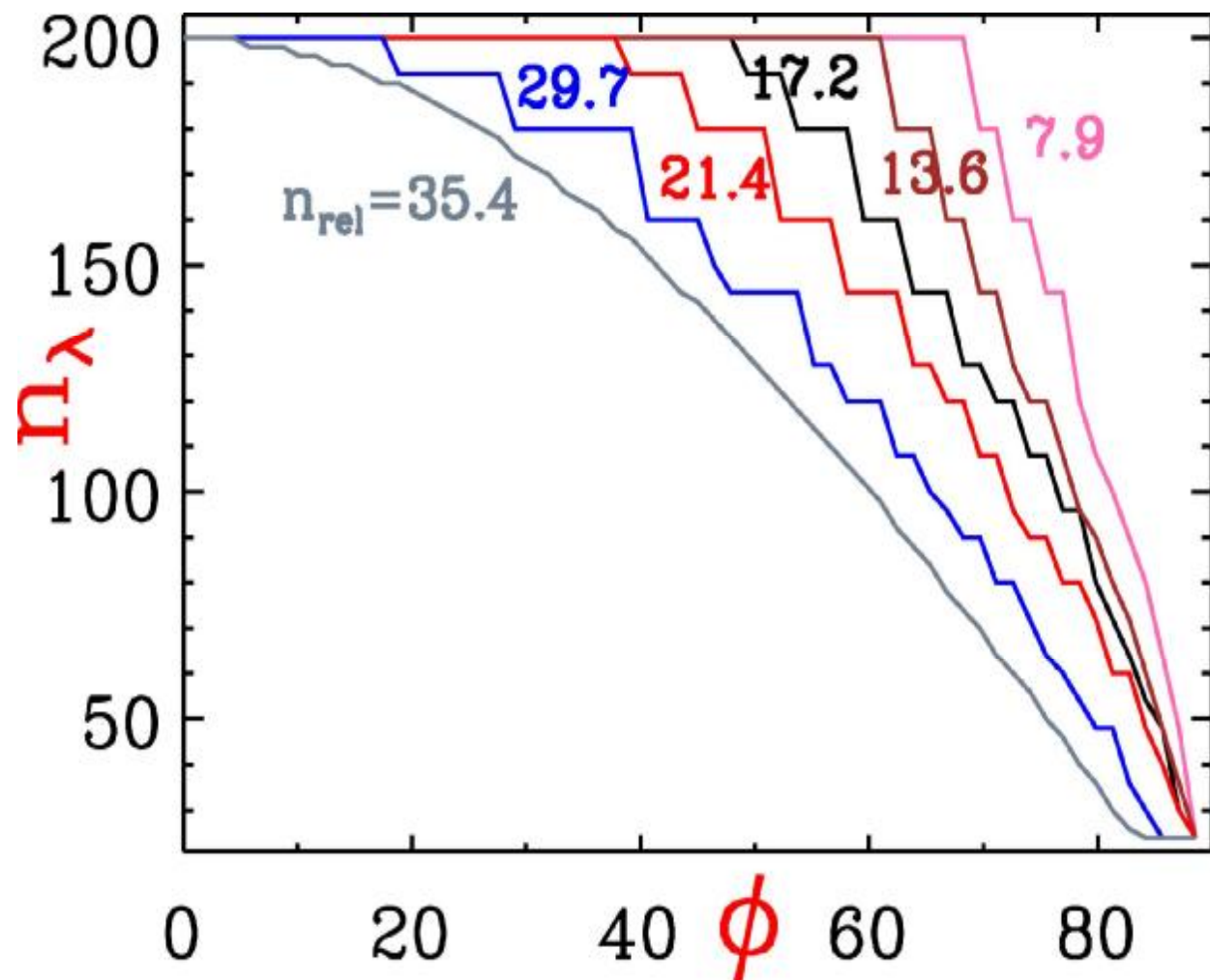
(Φ_0 is calculated on the regular grid)

$$\text{Criterion} \quad \int_0^{\pi/2} |\Phi - \Phi_0|\, d\phi_0 \Big/ \int_0^{\pi/2} \Phi_0\, d\phi_0 \le \epsilon_\Phi / 100.$$

ϵ_Φ	n_{rel}
0.00	0.0
0.01	7.9
0.05	10.5
0.20	13.6
0.50	15.3
1.05	17.2
2.00	19.2
4.00	21.4
21.6	29.7
50.8	35.4

n_{rel} is the relative reduction of the total number of nodes with respect to the regular grid
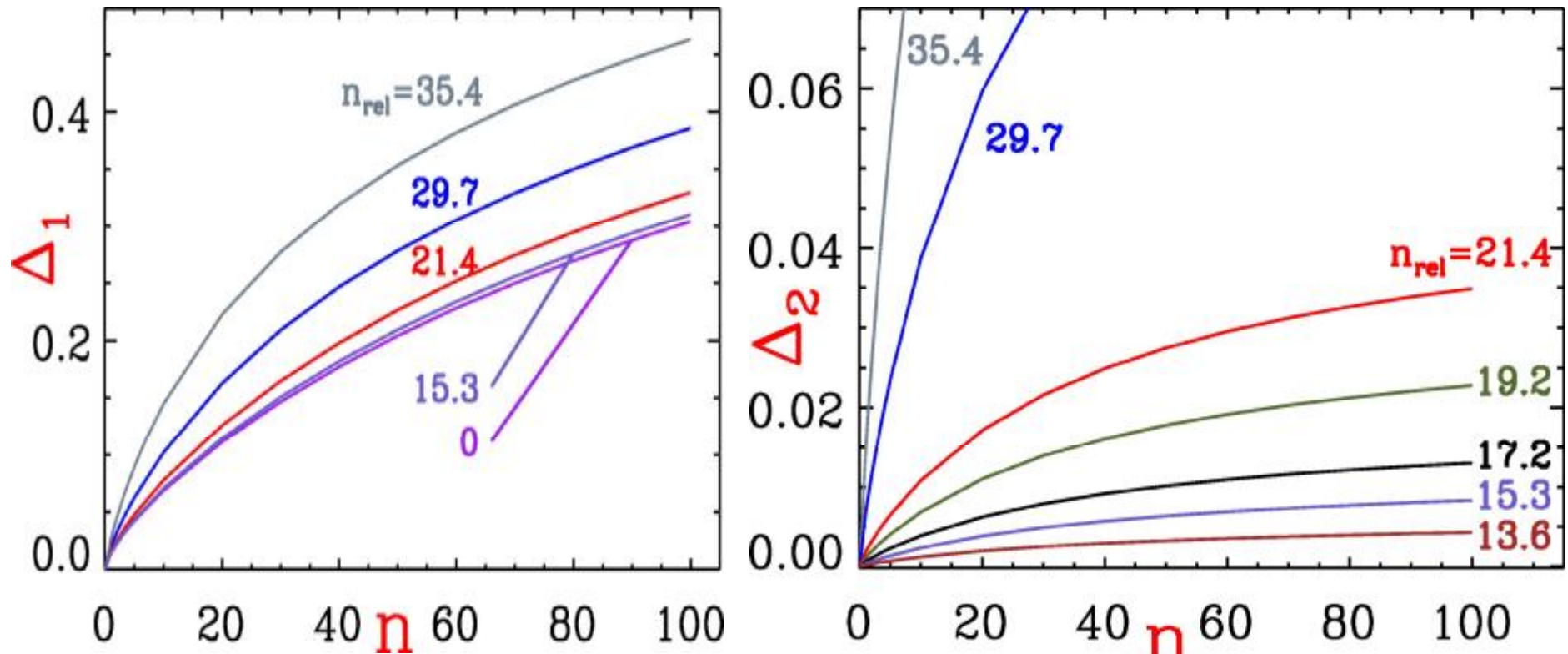
Solid body rotation test:

Williamson D. L. et al. - J. Comput. Phys., vol. 102, pp. 211-224.

The normalized r. m. s. error of the numerical solution with respect to analytical solution   **numerical solution obtained on the regular grid**
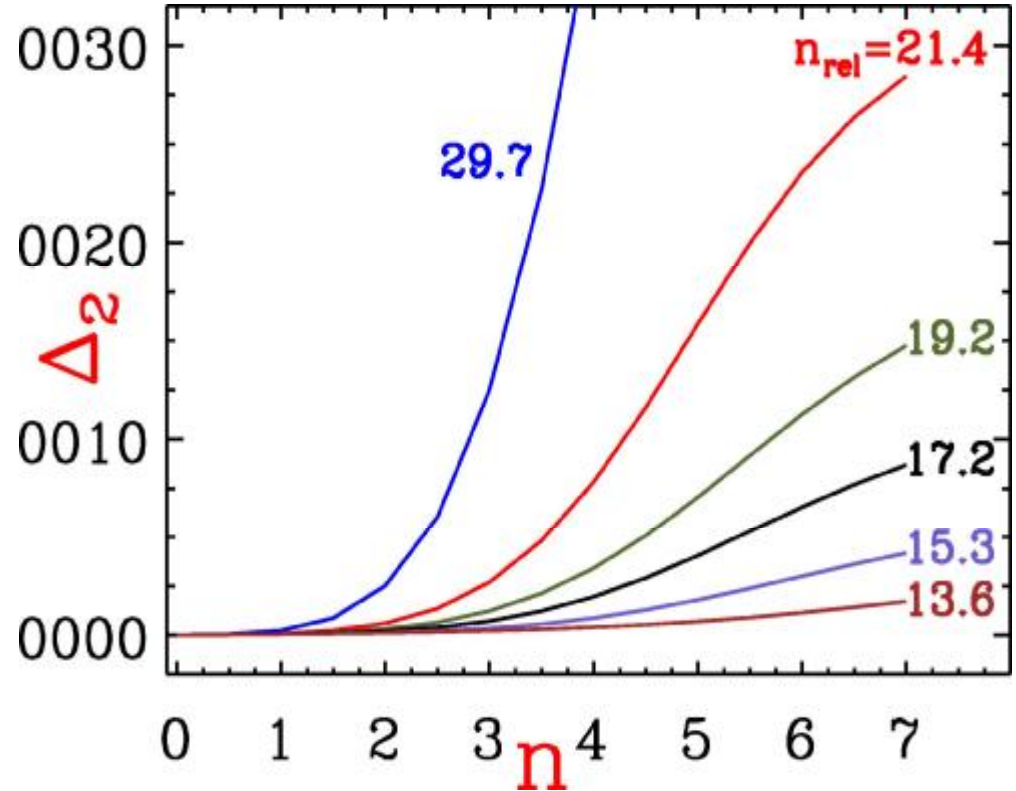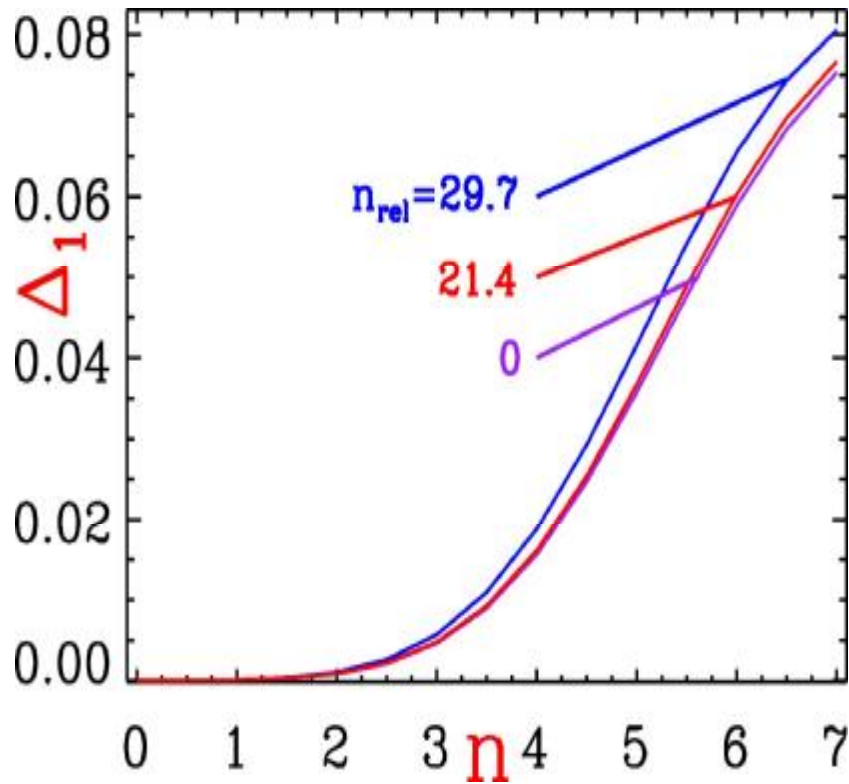

n is the number of rotations

Smooth deformational flow

Doswell S. A. - J. Atmos. Sci., 1984, vol. 41, pp. 1242-1248.
Nair R., et. al. - Mon. Wea. Rev., 2002, vol. 130, pp. 649-667.

The normalized r. m. s. error of the numerical solution with respect to
analytical solution   **numerical solution obtained on the regular grid**

n is the number of rotations

A reduced grid for the SL-AV global model

To appear in *Russian Meteorology and Hydrology*, 2006, N9

n is the number of rotations

Results from quasioperational tests at Hydrometcentre for Dec 2004-Aug 2005

Models compared here:

- **SMA** – Eulerian spectral T85 model with 31 levels, initial data – operational OI analyses.

- **SLM** – Semi-Lagrangian constant resolution SL-AV model (0.9°x0.72°, 28 levels), initial data – OI data assimilation based on this model (Tsyroulnikov et al, Russian Meteorology and Hydrology, 2003).

- **SLMV** – variable resolution SL-AV, 30 km over Russia, initial data – interpolation from SLM initial data (*adds 3-4 m of RMS error at H500 already at initial time*)

Verification – against operational OI analyses on 2.5° grid (favors SMA in data-sparse areas).
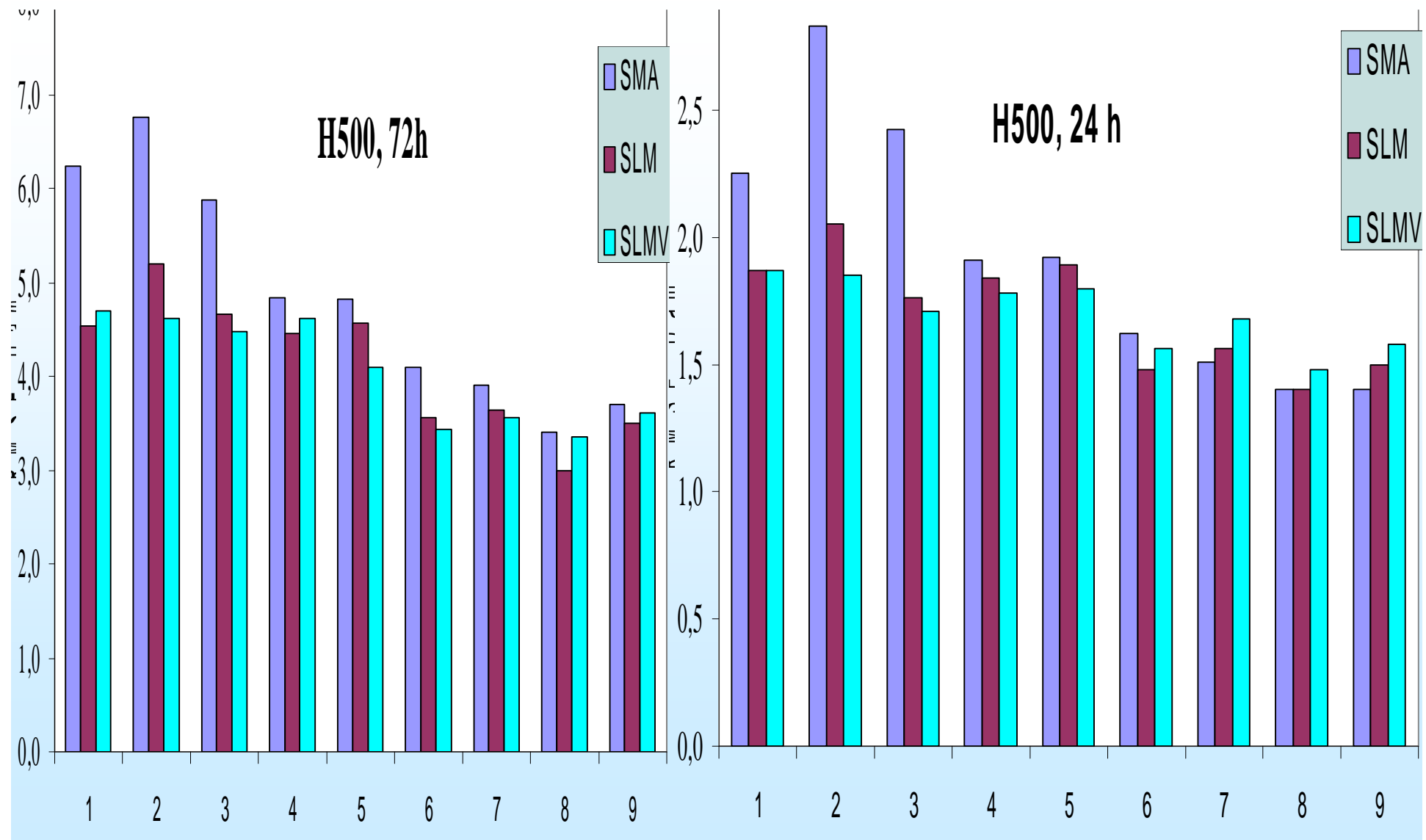
Monthly mean S1 H500 of 24 and 48h forecasts. Dec 2004 - Aug 2005. 12 UTC, Europe (verification against analyses)
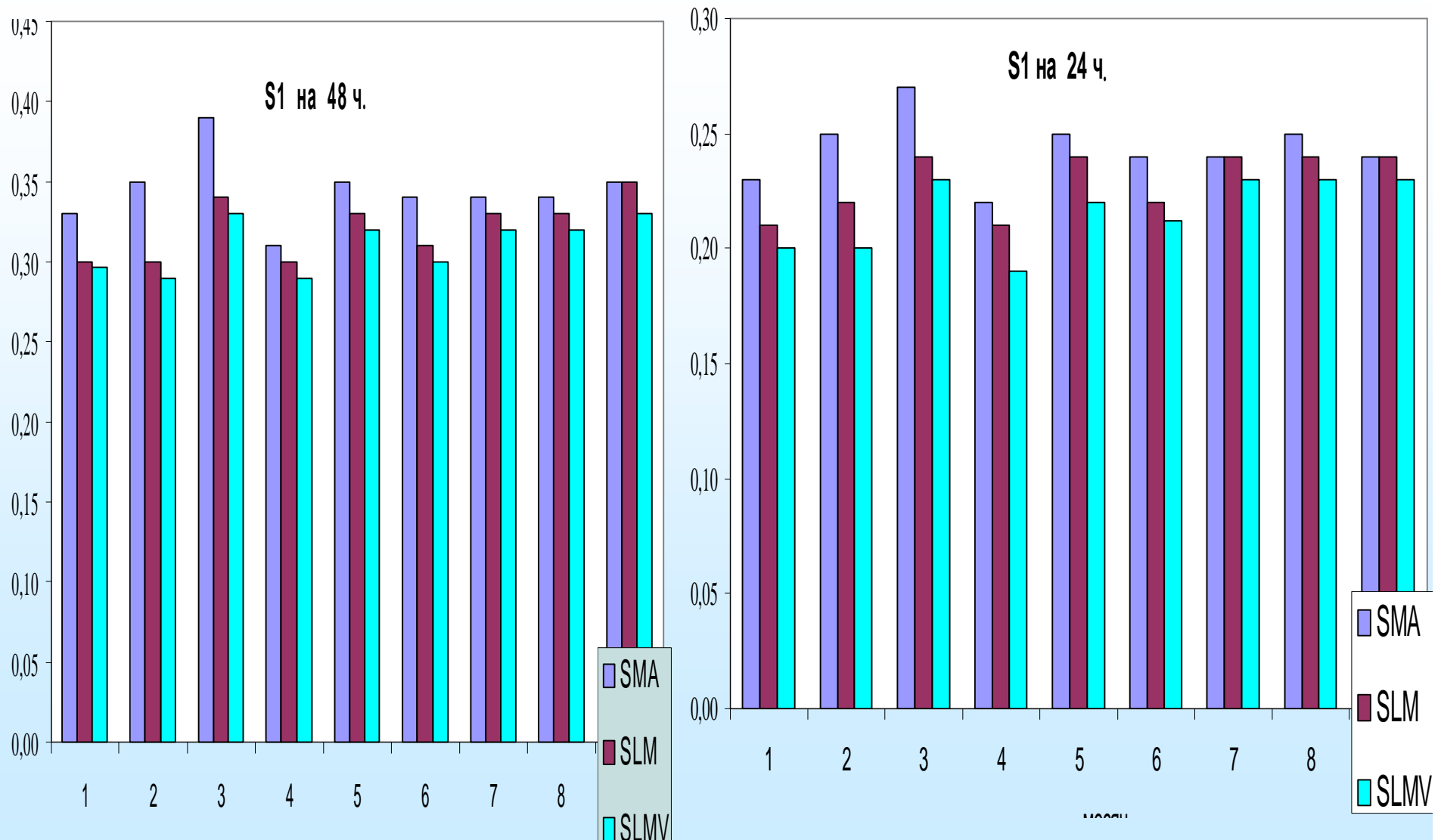
Monthly mean RMS errors of 24h and 48h T850 forecasts dec 2004 - aug 2005. 12 UTC, Europe. (verification against analyses)
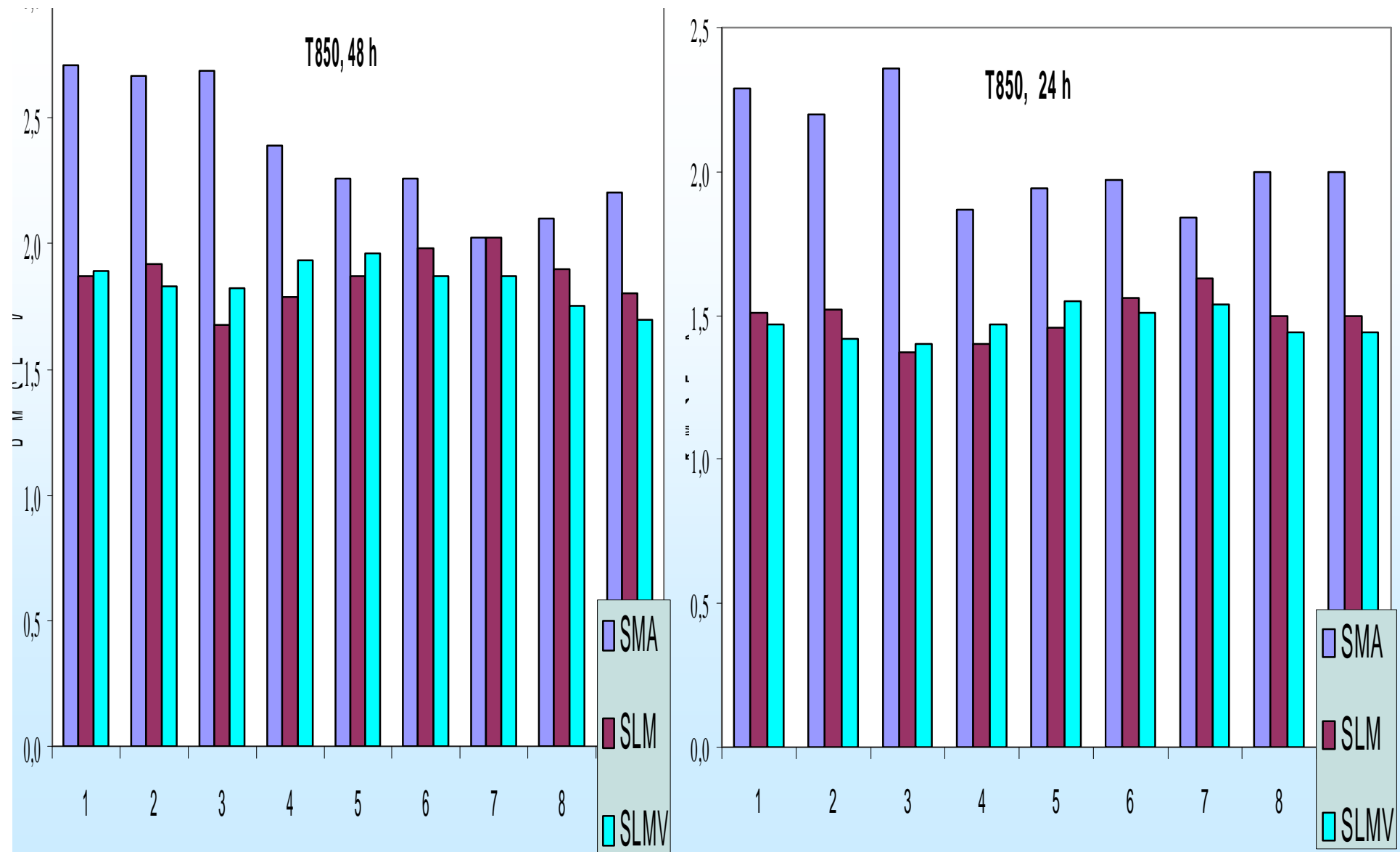
Monthly mean RMS errors of 24h and 48h H500 forecasts dec 2004 - aug 2005. 12 UTC, Europe. (verification against analyses)

Results for other regions and fields

- In Europe, variable-resolution SLAV model gives scores close to the constant-resolution version.

- In Asia, it produced somewhat worse scores.

- Advantage of the constant resolution SLAV version over SMA in Asia was less pronounced: the scores for some fields at 24 and 48 were better for SMA. This was changed after improvement of SLAV in Oct 2005 and verified during Nov 2005-May 2006.

Consequences

- **Constant resolution version** of SLAV is accepted as operational on 27/01/2006 for prediction of fields at P-levels and MSLP. Operational tests of precipitation forecasts have just started.

- **Variable resolution version** needs better initial data, at least 'poor man assimilation'. It is scheduled for operational tests again – this time including precipitation and near-surface temperature forecast

Evaluation of precipitation forecasts over Central Federal district of Russia for

May 2006

- Two versions of MM5 running at Russian Hydrometcentre and Moscow Hydrometeobureau (12-18 km resolution) and variable-resolution version of SL-AV model (30 km resolution) compared.

- Should *not* be considered as a judgement (period is too short), but rather as a demonstration of capabilities for SLAV model

Probability of detection (POD)
Предупрежденность осадков
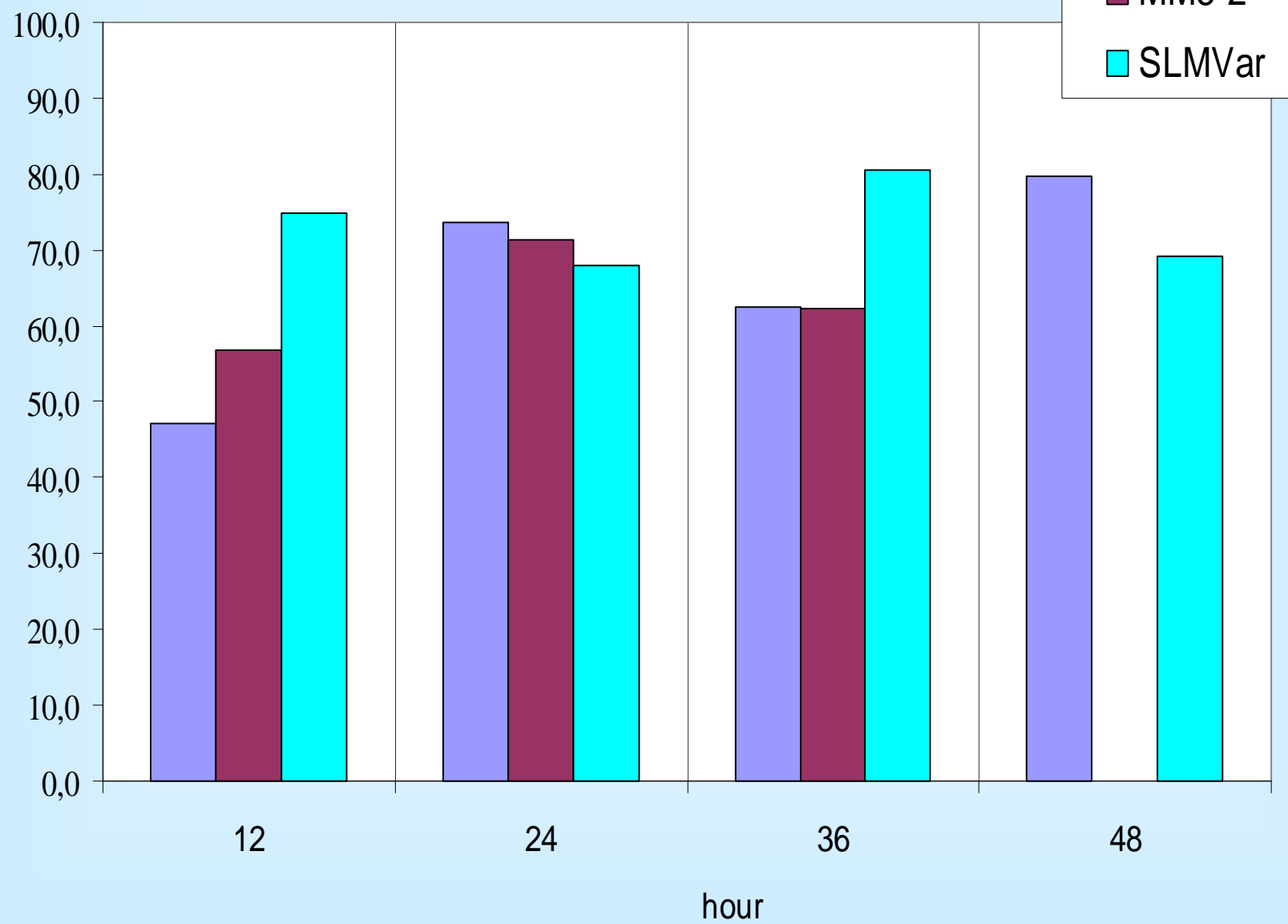
**Proportion correct
Общая оправдываемость
CFO**
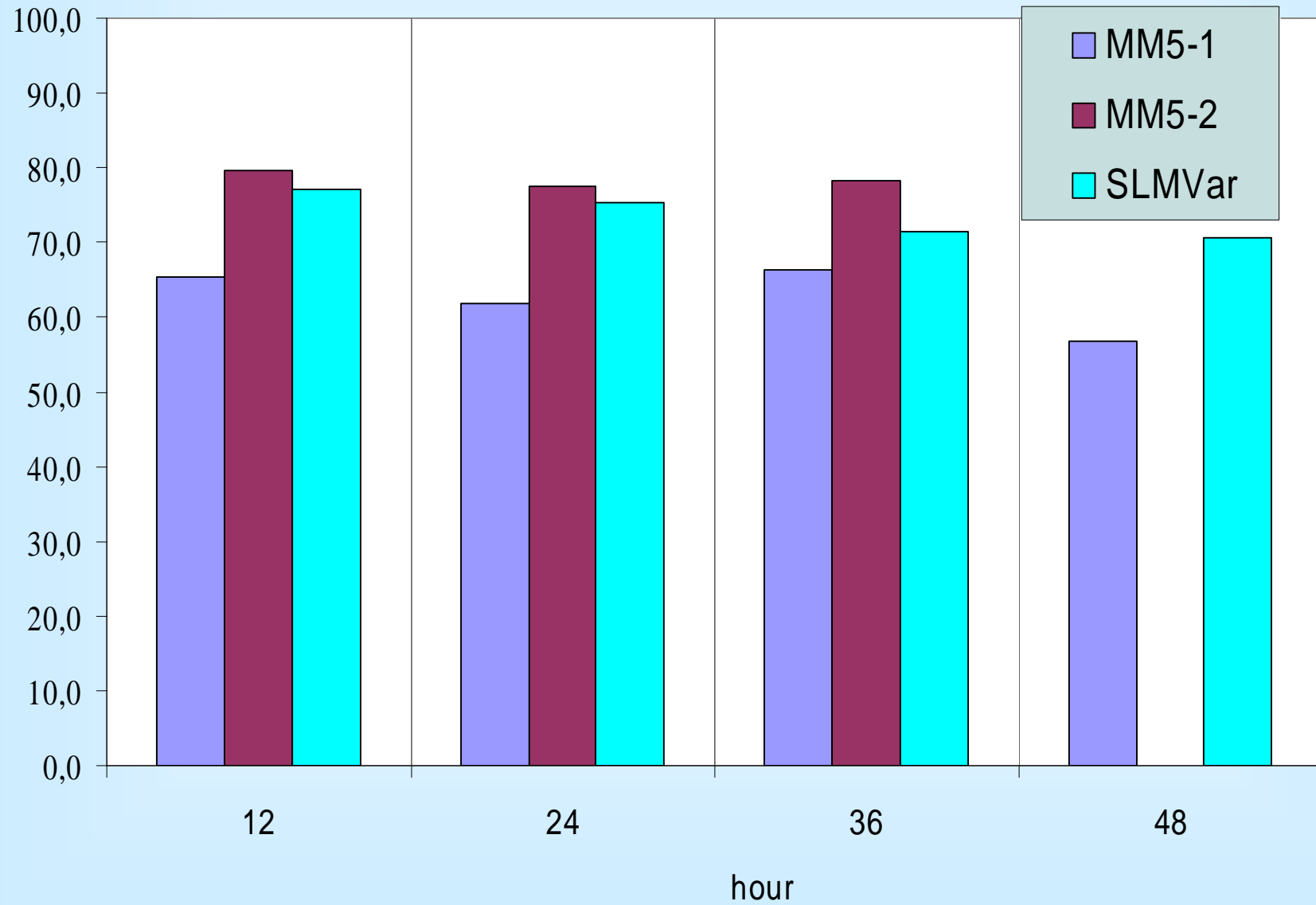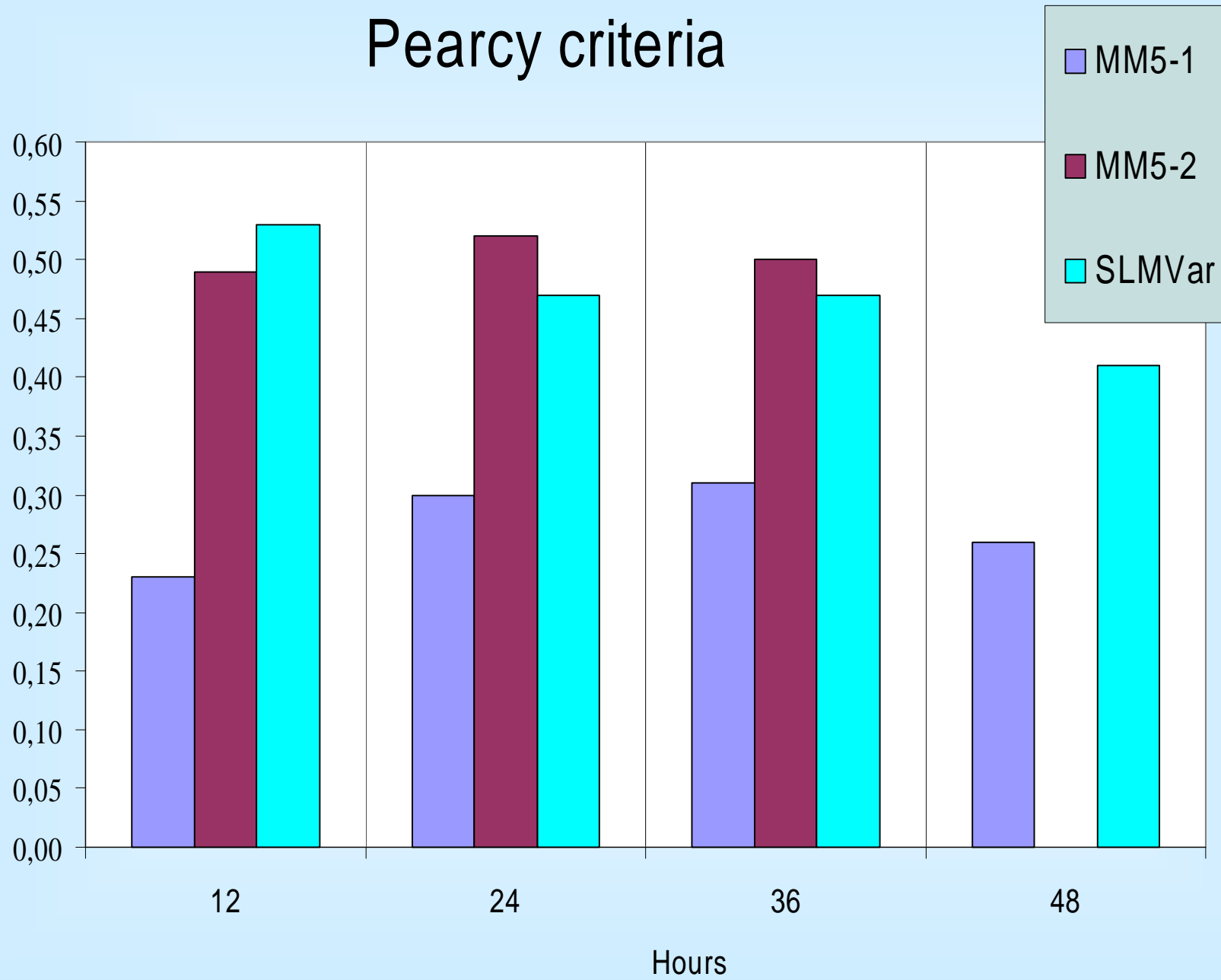
HSS (Heidke skill score)
(determines advantage of the forecast over random one)
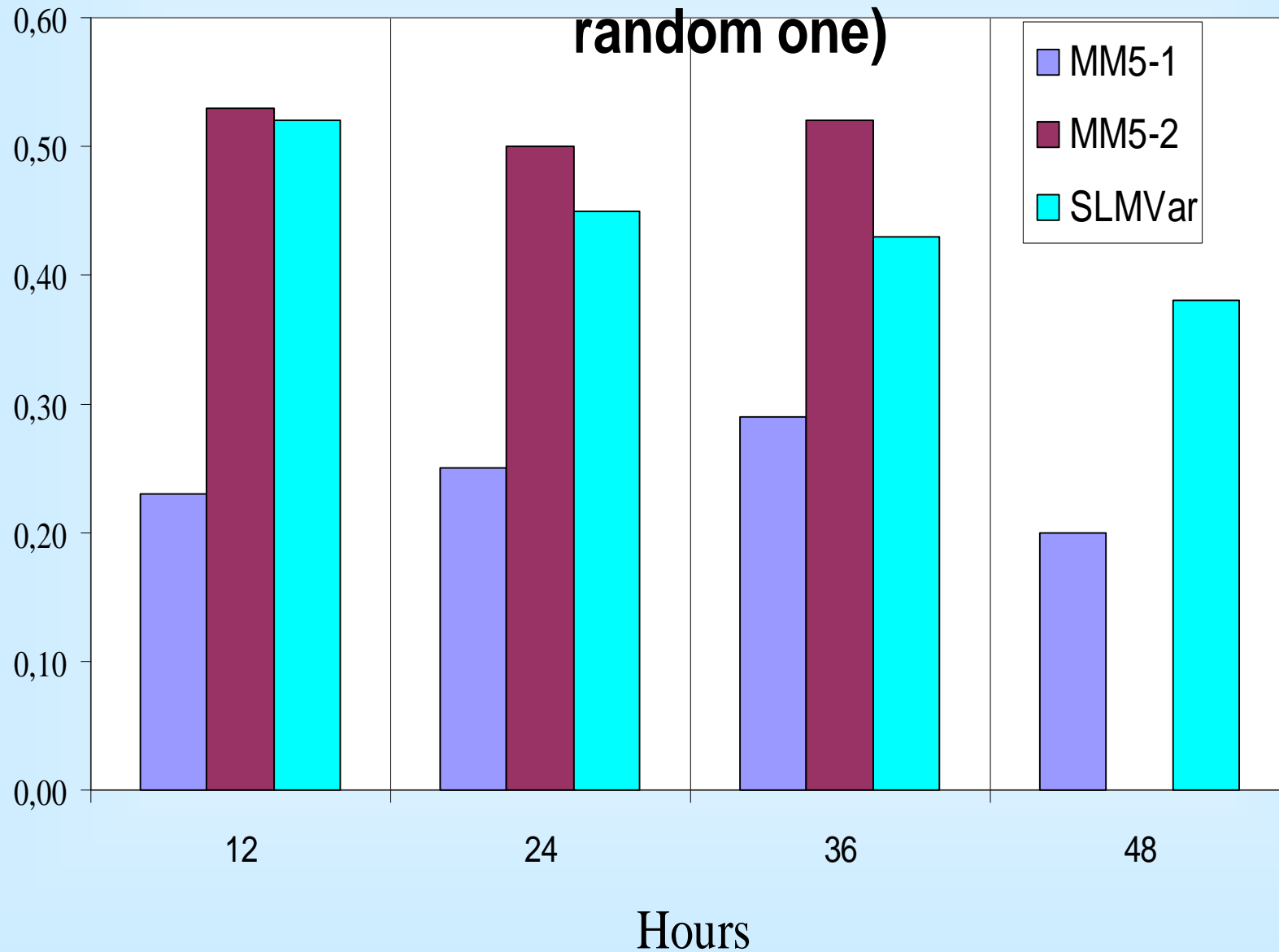
Precipitation forecast with var-res SLAV

- Variable-resolution SL-AV can give a competitive precipitation forecast at quite reasonable computational cost.

- Improvement of initial data, further tuning, and scheduled model developments should improve precipitation forecast.

Development of assimilation for soil variables

- Soil variables are not analyzed directly but are necessary as initial data for the model.

- Based on analysis of temperature and relative humidity at 2m according to *Giard, Bazile, MWR , 2000.*

- Presented in poster session by Nickolay Bogoslovskii.

Development of nonhydrostatic dynamical core - strategy

1. Development of 2D in vertical plane dynamical core based on junction of SLAV and nonhydrostatic HIRLAM (Room et al, HIRLAM Tech. Rep. N65). Account for different horizontal grids (staggered vs unstaggered) and numerics.

2. 2D conventional tests for nonhydrostatic dynamics.

3. Development and testing of 3D nonhydrostatic dynamical core.

Currently, (1) and partially (2) are implemented.

Why nonhydrostatic HIRLAM?

- HIRLAM uses semi-Lagrangian semi-implicit approach as well, so far it is also a finite-difference model.

- High computational efficiency.

Assumptions

- 3D velocity divergence =0 (no sound waves)

- No nonhydrotatic effects at the surface

Modifications to original NH HIRLAM algorithm

1. Unstaggered grid in horizontal, hence velocity is obtained from horizontal divergence and relative vorticity.

2. Fourth-order instead of second-order numerics for approximation of derivatives in the horizontal plane.

3. Trapezoidal rule instead of midpoint rule for vertical integrals.

Semi-implicit approach

Model equation

$$\frac{\partial \chi}{\partial t} = \mathcal{M}(\chi)$$

Semi-implicit scheme (leapfrog example)

$$\frac{\delta \chi}{\delta t} = \mathcal{M}(\chi) + \mathcal{L}\left(\overline{\chi}^t - \chi\right)$$

Semi-implicit, semi-Lagrangian approach (SISL)

F – linear part, a – small non-linear part

$$\frac{d\psi}{dt} = F + a$$

$$\frac{\psi^{t+\Delta t} - \psi_*^t}{\Delta t} = \frac{1-\varepsilon}{2}F_*^t + \frac{1+\varepsilon}{2}F^{t+\Delta t} + \frac{1-\varepsilon}{2}a_*^{t+\Delta t/2} + \frac{1+\varepsilon}{2}a^{t+\Delta t/2}$$

Time extrapolation

$$a^{t+\Delta t/2} = 1.5a^t - 0.5a^{t-\Delta t}$$

Numerical experiments

> Motionless isothermic atmosphere

> Mountain waves

Boundary conditions:

- Rigid lid both at the bottom and at the top

- Davies relaxation at lateral boundaries

Digital filter initialization for initial data

Motionless isothermic atmosphere with orography

Number of timsteps **N = 600**, **d t = 30 c**   Resolution: **d x = 528 м.**, **d z = 496 м.**

Vertical velocity **ω** in p-coordinate system

max(**ω**)=0.00103
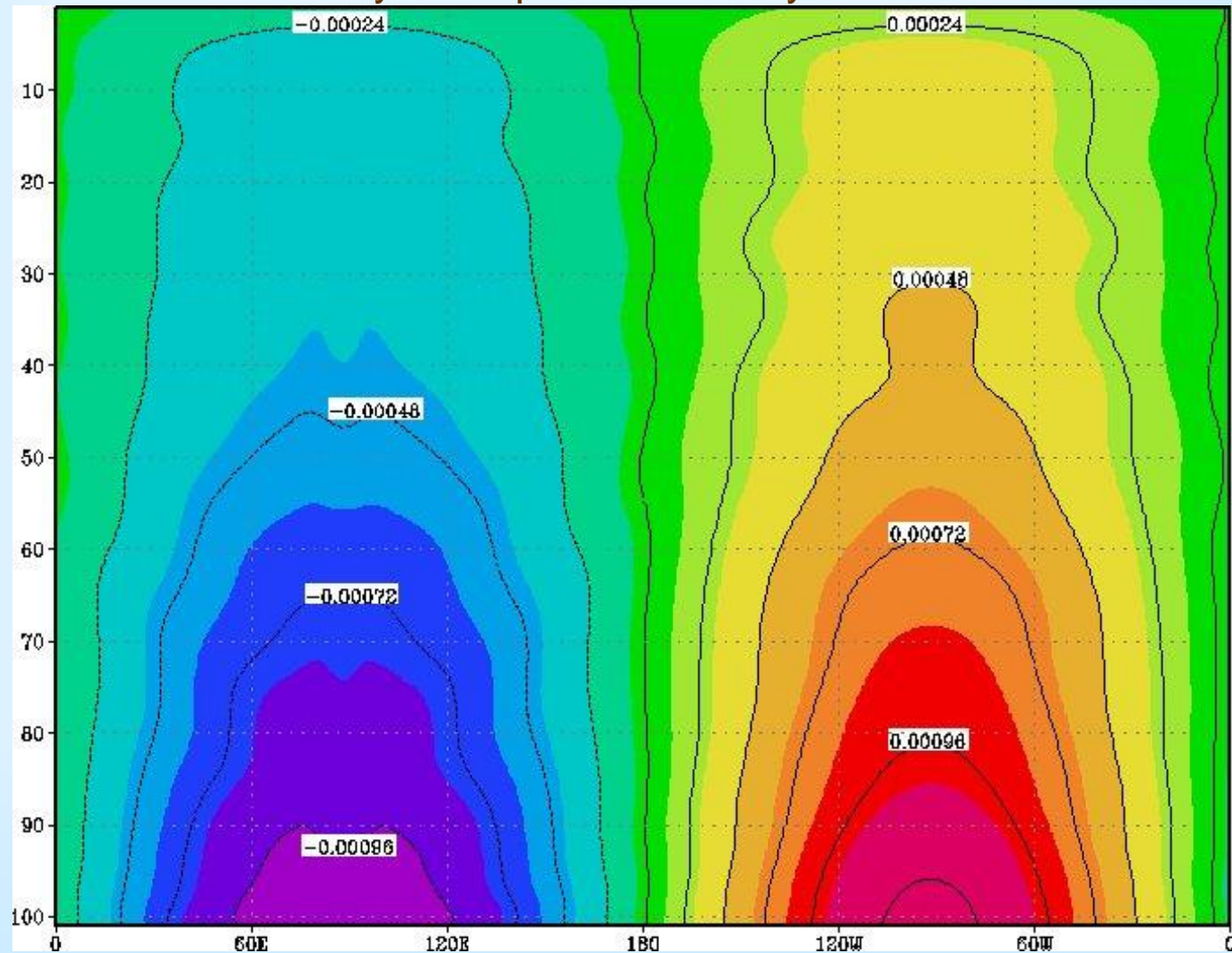
Mountain waves

Number of timesteps **N = 600**, **d t = 30 c**   Resolution: **d x = 528 м.**, **d z = 496 м.**

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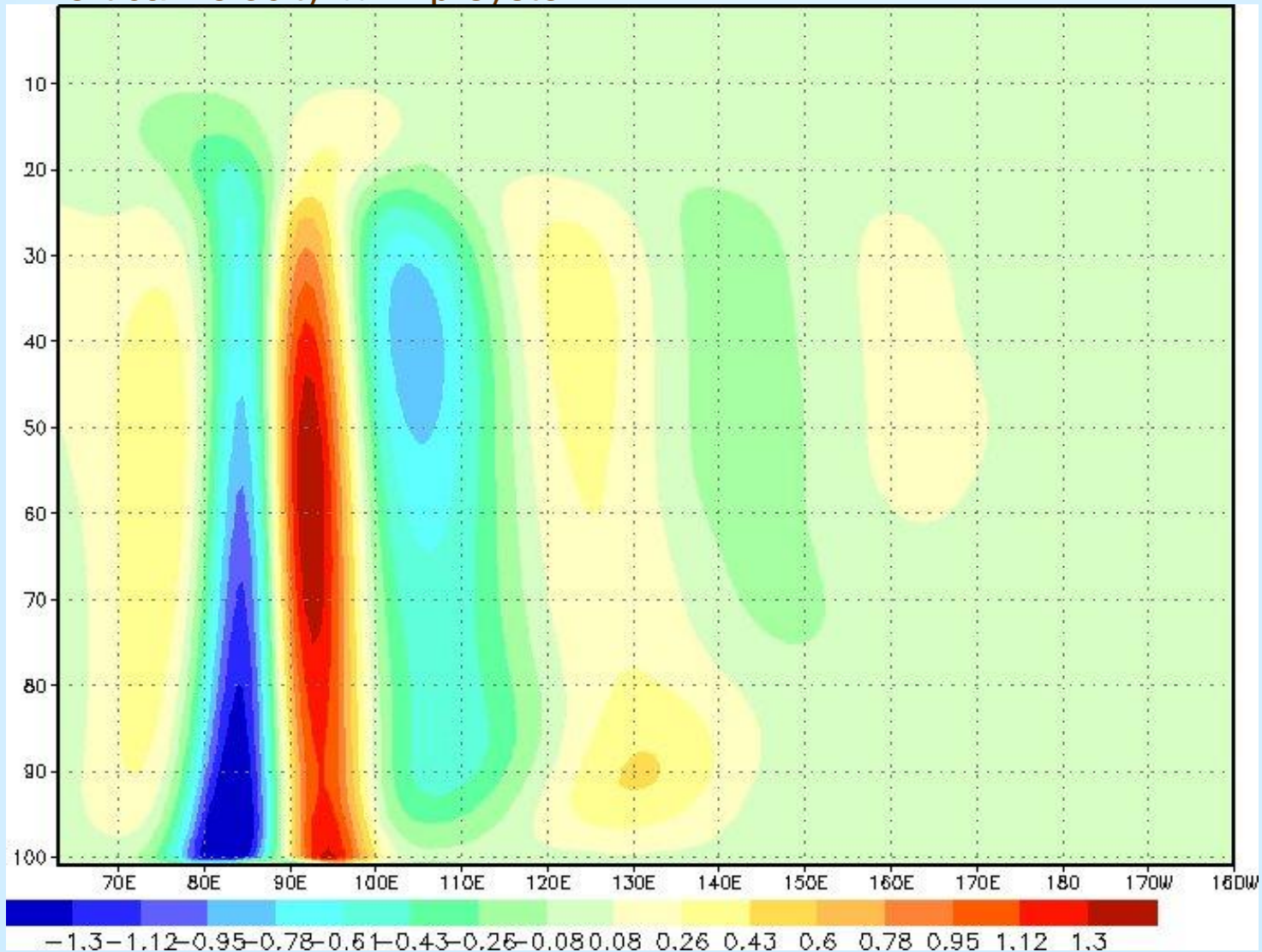
Future work

- **Further development and testing of a 2D nonhydrostatic version.**
- **Implementation of some sort of OI based assimilation for variable resolution SLAV model.**
- **Testing of already implemented finite-element scheme for vertical integrals in dynamics.**
- **Implementation of the reduced grid in complete variable resolution version of the SL-AV model.**

Hydrostatic model equations

$$\left(\frac{d\vec{U}}{dt} + f\,\vec{k} \times \frac{d\vec{r}}{dt} \right)_H = -\vec{\nabla}_\sigma \Phi - \frac{RT}{p_s}\vec{\nabla}p_s$$

$$\frac{\partial \Phi}{\partial \ln \sigma} = -RT$$

$$\frac{d \ln p_s}{dt} + D + \frac{\partial \dot{\sigma}}{\partial \sigma} = 0 \qquad\qquad \dot{\sigma} = \frac{d\sigma}{dt}$$

$$c_p \frac{dT}{dt} = RT \left(\frac{1}{p_s}\frac{dp_s}{dt} + \frac{\dot{\sigma}}{\sigma} \right)$$

Nonhydrostatic model equations

$$\left(\frac{\mathrm{d}\vec{U}}{\mathrm{d}t} + f\vec{k} \times \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}\right)_H = -\left(1 + \frac{1}{g}\frac{\mathrm{d}w}{\mathrm{d}t}\right)\vec{\nabla}_\sigma \Phi - \frac{RT}{p}\vec{\nabla}_\sigma p$$

$$\frac{\mathrm{d}w}{\mathrm{d}t} = g\left(\frac{\partial p}{\partial \pi} - 1\right) \qquad\qquad w = \frac{1}{g}\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

$$\frac{\mathrm{d}\ln p_s}{\mathrm{d}t} + D + \frac{\partial \dot{\sigma}}{\partial \sigma} = 0 \qquad\qquad p = \pi + p'$$

$$c_p \frac{\mathrm{d}T}{\mathrm{d}t} = \frac{RT}{p}\frac{\mathrm{d}p}{\mathrm{d}t} \qquad\qquad \frac{\partial \Phi}{\partial \pi} = -\frac{RT}{p}$$

Analytical model equations

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = F_\omega - \left(\frac{g\mathrm{p_s}\sigma}{RT}\right)^2 \frac{\partial\phi}{\mathrm{p_s}\partial\sigma} + \frac{c_v\omega^2}{c_p\mathrm{p_s}\sigma}$$

$$\frac{\mathrm{d}\mathbf{U}}{\mathrm{d}t} = \mathbf{F}_u - \hat{G}(\phi + \varphi) - \mathbf{f} \times \mathbf{U}_k$$

$$\frac{\mathrm{d}T'}{\mathrm{d}t} = F_T - \frac{\kappa T}{\mathrm{p_s}\sigma}\omega,$$

$$\frac{\mathrm{d}\chi}{\mathrm{d}t} + D + \frac{\partial\dot\sigma}{\partial\sigma} + \mathbf{U} \cdot \nabla(\ln \hat{\mathrm{p}}_s) = 0$$

$$\hat{G} \cdot \mathbf{U} + \frac{\partial\omega}{\mathrm{p_s}\partial\sigma} = 0 \qquad\qquad \hat{G} = \nabla_x - (\nabla(\ln \mathrm{p_s}))\sigma\frac{\partial}{\partial\sigma}$$

Semi-discrete model equations

$$\frac{d_{k+1/2}\omega}{dt} = -W_{k+1/2}(\phi_{k+1} - \phi_k) + (a_\omega^0)_{k+1/2}$$

$$\frac{d_k u}{dt} = -\hat{G}_k(\phi + \varphi)$$

$$\frac{d_k T'}{dt} = S_k \frac{\omega_{k+1/2} + \omega_{k-1/2}}{2}$$

$$\frac{d_k^h \chi}{dt} = -D_k - \frac{\dot{\sigma}_{k+1/2} - \dot{\sigma}_{k-1/2}}{\sigma_{k+1/2} - \sigma_{k-1/2}} - u_k \cdot \nabla(\ln \hat{p}_s)$$

$$\hat{G}_k(u) + \frac{\omega_{k+1/2} - \omega_{k-1/2}}{p_s(\sigma_{k+1/2} - \sigma_{k-1/2})} = 0$$

Nonlinear terms:

$$(a_\omega^0)_{k+1/2} = \omega_{k+1/2} \left(\frac{c_v \omega_{k+1/2}}{c_p p_s \sigma_{k+1/2}} \right)$$

$$W_{k+1/2} = \left(\frac{(g p_s \sigma)^2}{(RT)^2 p_s \Delta \sigma} ^{\sigma} \right)_{k+1/2}$$

$$S_k = \frac{\frac{R}{cp} T_k}{p_s(\overline{\sigma}_k^{\sigma})} - \frac{T_{k+1/2}^0 - T_{k-1/2}^0}{p_s(\sigma_{k+1/2} - \sigma_{k-1/2})}$$

Parallel implementation (MPI+OpenMP) 2

- Theoretical scalability is limited to N_{lat} ; for future 0.25°x0.18°x60 version this gives 1000 processors

- High efficiency of the code in single CPU mode: 21% from peak performance on scalar Itanium 2 1.3GHz CPU; ~50% on modern vector machines

- For 0.9°x0.72°x28 version, 24h forecast takes 2.5 min on INM RAS Myrinet 16 Itanium2 processor cluster